to *express* x as a function of y. Of course, we may have an indefinite number of defined ideas, and it is easy to see that this number does not affect the question as to how many indefinables there are.

For example, Whitehead and Russell, in their theory of deduction, have expressed *implication* and *joint assertion* of propositions in terms of *negation* and *disjunction*. Sheffer and Nicod have succeeded in defining all the above four ideas in terms of a new function of propositions p and q denoted by "p/q" and which can be defined in the system of Whitehead and Russell as "Either p or q is false." Thus, in the system of Whitehead and Russell we have five ideas of which two are indefinable, while in Sheffer's system we have five ideas of which one is indefinable. The latter is preferable because it discovers a relation between the two undefined ideas of Whitehead and Russell; though this relation does not appear capable, in our system of symbols, of being expressed without making use of Sheffer's indefinable. Thus the principle of parsimony appears, from a logical point of view, to be simply the maxim that logical analysis is to be carried as far as possible; and this is no more than Dedekind's maxim that what *can* be proved *is* to be proved.

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INDEPENDENCE PROOFS AND THE THEORY OF IMPLICATION.

In the January number Mr. Lenzen writes a criticism of the traditional symbolic logic as exemplified by the system of Whitehead and Russell's *Principia Mathematica*. His criticism is that it fails to give a correct theory of mathematical deduction. The attempt is made to prove that, in certain cases, of two mathematically independent postulates one implies the other according to the method of the *Principia*.

The present writer was, at first, in sympathy with the criticism; but finally the error involved was detected. The chief illustration is taken from two independent postulates for algebra:

$$\text{A2} \qquad (a+b)+c=a+(b+c),$$
$$\text{M5} \qquad a\times b=b\times a.$$

Applying the symbolic methods,

$$M5 \cdot \mathrm{c} \cdot A2 \,\mathrm{c}\, M5.$$

But, says the critic, M5 is true and therefore by the Principle of Inference (any proposition implied by a true proposition is true),

$$A2 \cdot \mathrm{c} \cdot M5.$$

The error of the criticism lies in asserting the truth of M5. The Principle of Inference requires M5 to be true universally; Huntington's proof of the independence of A2 and M5 involves, however, finding at least one case or system in which M5 is *not* true and in which A2 is true. Or we may put the matter in another form. If you once assert as true the postulates you are considering, you can no longer investigate their independence; because you cannot find a case in which one is false. On the other hand, if one is false, in some system, it is no longer true universally, and the Principle of Inference used by Mr. Lenzen has no support.

In fact, however, a deductive relation necessitates the use of at least one formal implication, which is a relation between propositional functions. Now it is, perhaps, a fair criticism of the *Principia* that it does not specify the various kinds of variable in terms of which a propositional function may be expressed. It may be a mathematical variable such as x or the shape of a triangle, or it may be a proposition, or it may be a situation or system in which it is asserted. This last view is unfortunately not mentioned by Whitehead and Russell, though the writer believes they would agree with it.

Let us return to the illustration given before. Call the system of algebra corresponding to Huntington's postulate, S_1. A2 and M5 are asserted for the System S_1, and we may call them

$$\begin{array}{ll} A(S) & (a+b)+c=a+(b+c), \\ M(S) & a\times b=b\times a, \end{array}$$

both asserted in the system S_1.

By definition A(S) formally implies M(S) only when

$$(S) : A(S) \cdot \mathrm{c} \cdot M(S);$$

that is:

$$(S) : \sim A(S) \cdot \mathrm{v} \cdot M(S).$$

But Huntington's analysis shows that, for some system, say S_2,

$$A(S_2) \cdot\wedge\cdot \sim M(S_2),$$

or

$$\sim[\sim A(S_2) \cdot \mathrm{v} \cdot M(S_2)].$$

That is to say, for this S_2, $A(S_2)$ does not imply $M(S_2)$. The formal implication falls to the ground.

Further on Mr. Lenzen says that a presumption is created that independence applies to propositions as well as to postulates. This presumption is probably an error, for any method of deduction involves the consideration of the particular propositions as special cases of propositional functions. If deductive relations could be found between propositions, these relations would also hold between the corresponding postulates, or else the propositions would not be typical cases. Contrary to Mr. Lenzen's assertion, it is possible sometimes to find deductive relations between particular propositions which correspond to independent postulates, if non-typical cases are chosen. The writer has found several but will choose one for the sake of illustration.

Consider the postulates

$$\text{(A)} \qquad a\,p\,b = b\,p\,a$$
$$\text{(B)} \qquad a\,p\,(b\,p\,c) = (a\,p\,b)\,p\,c$$

These are independent, for if $a\,p\,b$ means $(a+b)/2$, A is true while B is not; if $a\,p\,b$ means a, B is true while A is not; if $a\,p\,b = a\,b$, both are true.

Consider a particular case of B, namely,

$$a\,p\,(b\,p\,a) = (a\,p\,b)\,p\,a.$$

This can be deduced from (A) for

$$\begin{aligned} a\,p\,(b\,p\,a) &= a\,p\,(a\,p\,b) \\ &= (a\,p\,b)\,p\,a, \end{aligned}$$

using A twice.

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CAUSE AND EFFECT.

I.

The latest book by Mr. Bertrand Russell (*Mysticism and Logic, and Other Essays*, London and New York: Longmans, Green and Co., 1918; pp. viii, 234; price 7s. 6d. net) consists of reprints of ten previously (1901-1915) published papers which form a more or less complete whole and to which are added one or two notes written in 1917. The first essay, on "Mysticism and Logic," appeared in